1. Details of Module and its structure

Module Detail	
Subject Name	Physics
Course Name	Physics 02 (Physics part- 2, Class XI)
Module Name/Title	Unit 7, Module 2, Elastic Behaviour of materials Chapter 9, Mechanical Properties of Solids
Module Id	keph_20902_eContent
Pre-requisites	Materials are made up of atoms and molecules, internal forces are both attractive and repulsive.
Objectives	After going through this lesson, the learners will be able to: • **Distinguish** between: Elasticity and Plasticity • **Understand the reason** for Elastic and Plastic behaviour of materials • **Know** that a stress gets developed within materials in response to external deforming forces • **Express** strain as a measure of relative deformity • **State** Hooke's law • **Draw** Stress- strain curve for different materials and interpret them • **Use** Young's Modulus of Elasticity in relevant real life situations.
Keywords	Deforming force, elasticity, plasticity, tensile stress, longitudinal stress, tensile strain, longitudinal strain, Young's Modulus, elastic limit, yield point , fracture point, spring constant , load extension graph, elastomer

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1. UNIT SYLLABUS

UNIT 7: PROPERTIES OF BULK MATTER: 24 periods

Chapter–9: Mechanical Properties of Solids:

Elastic behaviour, Stress-strain relationship, Hooke's law, Young's modulus, bulk modulus, shear, modulus of rigidity, Poisson's ratio, elastic energy.

Chapter–10: Mechanical Properties of Fluids:

Pressure due to a fluid column; Pascal's law and its applications (hydraulic lift and hydraulic brakes). Effect of gravity on fluid pressure. Viscosity, Stokes' law, terminal velocity, streamline and turbulent flow, critical velocity, Bernoulli's theorem and its applications. Surface energy and surface tension, angle of contact, excess of pressure across a curved surface, application of surface tension ideas to drops, bubbles and capillary rise

Chapter–11: Thermal Properties of Matter:

Heat, temperature, thermal expansion; thermal expansion of solids, liquids and gases, anomalous expansion of water; specific heat capacity; Cp, Cv - calorimetry; change of state - latent heat capacity. Heat transfer-conduction, convection and radiation, thermal conductivity, qualitative ideas of Blackbody radiation, Wien's displacement Law, Stefan's law, Greenhouse effect.

2. MODULE-WISE DISTRIBUTION OF UNIT SYLLABUS 17 Modules

The above unit is divided into 17 modules for better understanding

Module 1	• **Forces between atoms and molecules making up the bulk matter** • **Reasons to believe that intermolecular and interatomic forces exist**

	• Overview of unit • State of matter • Study of a few selected properties of matter • Study of elastic behaviour of solids • Stationary fluid property: pressure and viscosity • Stationary liquid property: surface tension • Properties of Flowing fluids • Effect of heat on matter
Module 2	• Idea of deformation by external force • Elastic nature of materials • Elastic behaviour • Plastic behaviour • Tensile stress • Longitudinal Stress and longitudinal strain • Relation between stress and strain • Hooke's law • Young's modulus of elasticity 'Y'
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Module 5	• Shear strain • Shear stress • Modulus of Rigidity G • Poisson's ratio • Elastic energy • To study the effect of load on depression of a suitably clamped meter scale loaded at i)its ends ii)in the middle • Height of sand heaps , height of mountains
Module 6	• Fluids-liquids and gases • Stationary and flowing fluids • Pressure due to a fluid column • Pressure exerted by solid , liquids and gases • Direction of Pressure exerted by solids, liquids and gases
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	• Stokes' Law • Terminal velocity • Examples • Determine the coefficient of viscosity of a given viscous liquid by measuring terminal velocity of a given spherical body in the laboratory
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Module 12	• Thermal properties of matter • Heat • Temperature • Thermometers
Module 13	• Thermal expansion • To observe and explain the effect of heating on a bi-metallic strip

	• Practical applications of bimetallic strips • Expansion of solids, liquids and gases • To note the change in the level of liquid in a container on heating and to interpret the results • Anomalous expansion of water
Module 14	• Rise in temperature • Heat capacity of a body • Specific heat capacity of a material • Calorimetry • To determine specific heat capacity of a given solid material by the method of mixtures • Heat capacities of a gas have a large range • Specific heat at constant volume C_V • Specific heat capacity at constant pressure C_P
Module 15	• Change of state • To observe change of state and plot a cooling curve for molten wax. • Melting point, Regelation, Evaporation, boiling point, sublimation • Triple point of water • Latent heat of fusion • Latent heat of vaporisation • Calorimetry and determination of specific latent heat capacity
Module 16	• Heat Transfer • Conduction, convection, radiation • Coefficient of thermal conductivity • Convection
Module 17	• Black body • Black body radiation • Wien's displacement law • Stefan's law • Newton's law of cooling, • To study the temperature, time relation for a hot body by plotting its cooling curve • To study the factors affecting the rate of loss of heat of a liquid • Greenhouse effect

Module 2

3. WORDS YOU MUST KNOW

- **Rigid body:** is a solid body in which deformation due to external forces is zero or is so small that can be neglected. The distance between any two given points in a rigid body remains constant in time regardless of external forces exerted on it. A rigid body is usually considered as a continuous distribution of mass.

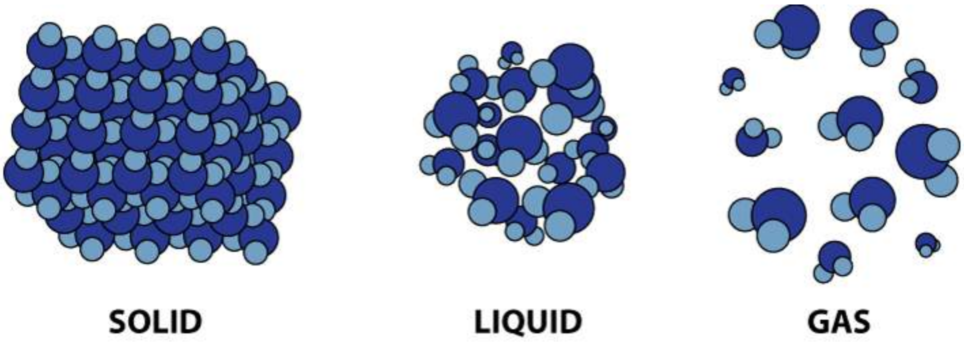
- **Inter-atomic forces:** are the **forces** which mediate interaction between atoms and molecules. These includes **forces** of attraction or repulsion which act between molecules and other types of neighbouring particles, e.g., atoms or ions.

- **Internal structure of Solid:**
  Crystalline solid: is a **solid** material whose constituents (such as atoms, molecules, or ions) are arranged in a highly ordered microscopic structure, forming a regular (geometry) crystal lattice that extends in all directions.
  Lattice: Ionic compounds are made up of ions - positive and negatively charged particles. These positive and negative ions attract each other and group together in quite large structures called lattices. In the lattice, each positive ion is surrounded by (several) negative ions.

- **Bond length:** In molecular geometry, bond length or bond distance is the average distance between nuclei of two bonded atoms in a molecule. It is a transferable property of a **bond** between atoms of fixed types, relatively independent of the rest of the molecule.

- **Bond energy:** bond energy (E) or (bond enthalpy (H)) is a measure of bond strength in a chemical bond.
- **Amorphous solid:** or non-crystalline solid is a solid that lacks the long-range order that is characteristic of a crystal. The term has often been used synonymously with glass.
- **Molecular structure of Liquid:** A **liquid** is a nearly incompressible fluid that conforms to the shape of its container but retains a (nearly) constant volume independent of pressure. As such, it is one of the three fundamental states of matter (the others being solid and gas), and is the only state with a definite volume but no fixed shape. A liquid is made up of tiny vibrating particles of matter, such as atoms, held together by interatomic/intermolecular bonds.
  Water is, by far, the most common liquid on Earth. Like a gas, a liquid is able to flow and take the shape of its container. Most liquids resist compression, although some can be compressed. Unlike a gas, a liquid does not disperse to fill every space of the container, and maintains a fairly constant density. A distinctive property of the liquid state is surface tension, leading to wetting phenomena also called capillarity..

- **Molecular structure of gases: Gas** is one of the three fundamental states of matter (the others being solid and liquid). A pure gas may be made up of individual atoms as in Argon or Neon , molecules made from one type of atoms like hydrogen, oxygen or compound molecules made from a variety of atoms like water. A gas mixture can contain a variety of pure gases much like the air. What distinguishes a gas, from liquids and solids, is the vast separation of the individual gas particles. This separation usually makes a colourless gas invisible to the human observer.



https://en.wikipedia.org/wiki/State_of_matter

4. INTRODUCTION

A rigid body generally means a hard solid object having a definite shape and size. However, in reality, all bodies can be stretched, compressed and bent. Even the appreciably rigid steel bar can be deformed when a sufficiently large external force is applied on it.You may have seen iron rods at construction sites .

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This means that all practical **solid bodies** are never perfectly rigid.

A solid has a definite shape and size. In order to change (or deform) the shape or size of a body, a **force** is required. Such a force is called a **deforming force.** If you stretch a helical spring by gently pulling its ends, the length of the spring increases slightly. When you leave the ends of the spring, it regains its original size and shape.

We have experienced the use of such a spring in some pens and pencils. Pressing the pen cap allow the writing point to come out of the pen casing (with a click sound). On pressing it again the writing point goes back inside the pen/ pencil casing.

Two points are to be noted here

- **The spring inside the pen gets deformed on pushing the pen cap.**
- **It regains the original shape once the deforming force is removed.**

The property of a body, by virtue of which it tends to regain its original size and shape when the applied (deforming) force is removed, is known as elasticity, the **deformation** caused is known as **elastic deformation**.

However, if you apply force to a lump of putty, potters clay or dough for making roti, they have no gross tendency to regain their previous shape; they tend to get permanently deformed.
Such substances are called **plastic** and this property is called **plasticity**. Putty and clay are close to being ideal plastics.

All materials are elastic up to a certain deformation; then they turn plastic. Springs lose their springiness if subjected to large external force and tend to become 'plastic'.

The elastic behaviour of materials plays an important role in engineering designs.

For Example:

- **In order to design a building, knowledge of elastic properties of materials (like steel, concrete etc.) is essential.**
- **The design of bridges, automobiles, ropeways etc., has to take into account the elastic properties of its materials.**
- **One could also ask — Can we design an aeroplane which is very light but sufficiently strong?**
- **Can we design an artificial limb which is lighter but stronger?**
- **Why does a railway track have a particular shape (like I)?**
- **Why is glass brittle while brass is not?**
- **Are solids more compressible then liquids?**
- **If we take water in a plastic bag then it takes the shape of bag, now we apply a force and the shape of water is deformed and when this force is removed water regains its shape, can this property of water be called elasticity?**

In an elastic body when we apply a force as in a spring then this force is incurred by which of the following

Bonds between the atoms
Force of attraction between atoms
Binding energy of the atoms
Elastic nature of atoms

Answers to such questions begin with the study of how relatively simple kinds of loads or forces act to deform different solid bodies. We need to also study the common materials in which change in shape, volume may take place whenever **external deforming forces** are applied to them.

In this unit, we shall study the elastic behaviour and mechanical properties of solids; this would answer many such questions.

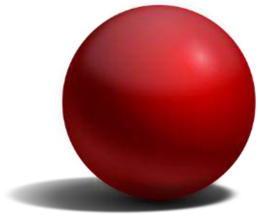
IDEA OF DEFORMATION BY EXTERNAL FORCE:

In mechanical properties of solids, **deformation refers to any changes in the shape or size of an object due to an applied force.**

Why do we need to know about the elastic nature of a material?

- **What it means to be Elastic?**

If we squeeze a rubber ball and release it, it (almost) gets restored to its original shape. **However, if we squeeze an orange, it would not regain its original shape.**



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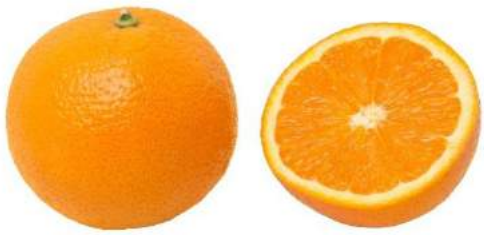

https://upload.wikimedia.org/wikipedia/commons/7/7b/Orange-Whole-%26-Split.jpg

A metal chair regains its original shape and size but a juice can (made of tin)? it would not regain its original shape when the deforming force is removed.

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what would happen to a tube of tooth paste, when you put some of the paste on your brush every morning ?

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THINK ABOUT THESE

- **Can you bend a glass rod? It will break because glass is more brittle than elastic.**

- **An athlete scales a high jump using a pole vault , What would happen in case of a wooden rod?**

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- **What about our bones, muscles and teeth?**

SOME EXAMPLES WE SEE AROUND US.

- A **rubber band** gets stretched when we apply force on it. However, it (almost) regains the original shape once the force is removed.

But does it always come back to its original shape and size irrespective of the magnitude of force?

- A **slingshot (gulel, catapult)** used to shoot a body at high speed involves the uses of specially made rubber bands. The body, placed at the centre of the band, is drawn back as far as possible and released, thus striking the target with a great impact.

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- Similar is the case of **bowstring,** stretched back with the arrow to release it with high speed.

If we replace the rubber used in the above examples with a cotton cord, would they still work in the same way?

In order to change (or deform) the shape or size of a body, a force is required.

For example, if you stretch a helical spring by gently pulling its ends, the length of the spring increases slightly. When you leave the ends of the spring, it regains its original size and shape.

The property of a body, by virtue of which it tends to regain its original size and shape when the applied force is removed, is known as elasticity; the deformation caused is known as elastic deformation.

We usually consider rigid bodies - A rigid body means a hard solid object having a definite shape and size. But in reality, bodies can be stretched, compressed, bent and twisted.

Even the rigid steel bar can be deformed when a sufficiently large force is applied on it. This means that solid bodies are not perfectly rigid.

Can rigid bodies be elastic?

Do solids, liquids and gases, all possess the property of elasticity?

Liquids and gases also possess elastic properties since their volumes changes with the application of pressure. i.e. both liquids and gases are compressible. **(We will consider this in the succeeding modules)**
In this module we will only consider elasticity of solids

5. ELASTIC BEHAVIOUR OF SOLIDS

The elastic behaviour of materials plays an important role in engineering design.
For example, while designing a flyover, knowledge of elastic properties of materials, like steel, concrete etc., is essential.

The gates of a building, rope used in crane to lift heavy objects, the almirahs and the safe lockers are all made of steel for a reason. The same is true in the design of bridges, automobiles, ropeways, cable cars etc.

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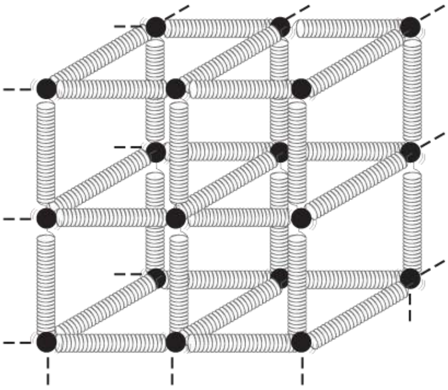
https://upload.wikimedia.org/wikipedia/commons/f/f2/Jawaharlal_Nehru_Stadium_Delhi_outside_view%2C_July_2016.jpg

https://upload.wikimedia.org/wikipedia/commons/9/9d/Chhatarpur_Metro_station_CDR_Chowk.jpg

We know that in a solid, each atom or molecule is surrounded by neighbouring atoms or molecules. These are bonded together by inter-atomic or intermolecular forces and stay in a stable equilibrium position. When a solid is deformed, the atoms or molecules are displaced from their equilibrium positions causing a change in the inter-atomic (or intermolecular) distances. When the deforming force is removed, the inter-atomic forces tend to drive them back to their original positions. Thus the body regains its original shape and size. The atoms/molecules are bounded together by inter-atomic or intermolecular forces which tend to make them stay in a stable equilibrium position.

Consider a spring ball system as shown. If you try to displace any ball from its equilibrium position, the spring system tries to restore the ball back to its original position.



The spring ball system is used to explain the behaviour of a solid under the application of a deforming force.

When a solid is deformed, the atoms or molecules are displaced from their equilibrium positions causing a change in the inter-atomic (or intermolecular) distances. When the deforming force is removed, the inter-atomic forces tend to drive them back to their original positions. Thus, the body regains its original shape and size (similar to what we see in the case of a spring).

PLASTIC BEHAVIOUR
How is Plasticity different from Elasticity?

If you apply force to a lump of putty, potters clay or play-dough, they have no tendency to regain their previous shape; they get permanently deformed.

https://cdn.pixabay.com/photo/2016/05/15/19/02/dough-1394214_960_720.jpg

Such substances are called plastic and the property is called plasticity. Putty and mud are close to ideal plastics.

Think about the socks you wear. Initially the 'elastic' holds the socks up on your feet; with constant wear after some time the elastic loses its properties and is unable to keep the socks up. It then tends to become plastic in nature.

https://c1.staticflickr.com/3/2509/4202477346_3a38524232_b.jpg

- If we want to build a cable-bridge then the metal used for cable should be more elastic or less elastic?
- Does the property of elasticity of a material is different for its different shapes?
- In the plastic behaviour why the interatomic forces do not tend to bring the atom to its earlier position?
- Is the property of elasticity and plasticity applicable to solids only ?

6. HOW DO WE MEASURE ELASTICITY?

Elasticity is a measure of how much an object deforms (gets **strained**) when a given **deforming force** is applied.

WHAT IS STRESS?

When a body is subjected to a deforming force, an **internal restoring force** is developed in the body.
We need to recall our inter-atomic force –inter-atomic separation graph.

The internal force opposes the deforming force. It is self-adjusting in magnitude (*the restoring force is equal to the deforming force*), and direction (restoring *force is opposite to the deforming force*) and nature *(if the inter-atomic separation is increased by the deforming force the restoring force is due to the internal forces of attraction, but in case the inter-atomic separation decreases, due to the deforming forces, the restoring force is due to repulsion between atoms*)

Thus **restoring force is equal in magnitude but opposite in direction to the applied deforming force.**

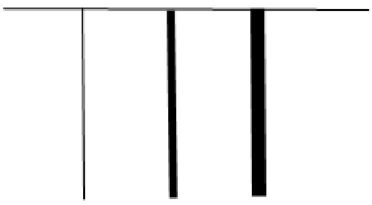
The restoring force, developed per unit area, is known as stress.

If F is the restoring force and A is the area of cross section of the body,

Magnitude of the stress = $\frac{F}{A}$

The SI unit of stress is N m^{-2} or pascal (Pa) and its dimensional formula is [ML^{-1}T^{-2}].

EXAMPLE



The figure shows three wires of the same material but different area of cross section. Same deforming force is applied to each of the wires by hanging the same load at its free end.

- **i) What is the direction of deforming force in each case?**
- **ii) What is the direction of stress in each case?**
- **iii) The deforming force (external load) is the same for the three cases, Is the stress equal in all cases?**
- **iv) In which wire is the stress maximum?**
- **v) In which wire is the stress minimum?**

SOLUTION

i) downward along the wire

ii) upward along the wire

iii) no, because area of cross section of wires is not the same

iv) stress = F/A, the thinnest wire will have maximum stress

vi) The thickest wire will have the least stress

THINK ABOUT THESE

- **Is restoring force always equal to the deforming force?**

- **When we apply a deforming force then there is an internal restoring force which is equal and opposite to the deforming force. Is it an example of Newton's third law?**
- **There are two wires of 5 and 10 cm. the area of cross-section of each wire is same. if we apply the deforming force along the length of wires then value of stress in each wire will be-**

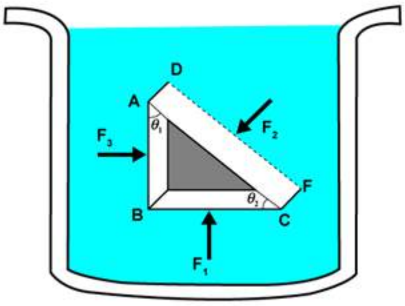
  Equal in both wire
  Greater in small wire
  Greater in long wire
  Can't be compared
- **Is there a 'limit of elasticity' for all materials?**
- **Is the property of development of stress only for solids?**

This is not 'examination stress' which is a state of mental or emotional tension resulting from adverse or demanding circumstances.

"I studied hard and prepared well for the examination but felt 'washed out' in the examination"

"He's obviously under a lot of stress"

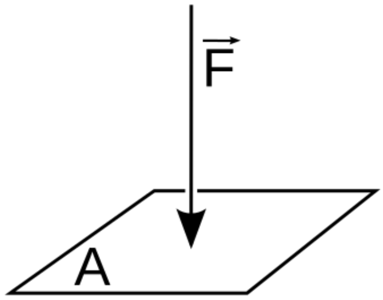
DIFFERENCE BETWEEN STRESS AND PRESSURE



Picture 1

Picture 2

Pressure (symbol: *p* or *P*) is the force applied perpendicular to the surface of an object per unit area over which that force is distributed.



https://upload.wikimedia.org/wikipedia/commons/thumb/f/ff/Pressure_force_area.svg/350px-Pressure_force_area.svg.png

$$\text{pressure} = \frac{\text{force or thrust}}{\text{area}}$$

THINK ABOUT THESE

- **Why is pressure is a scalar quantity even though force causing it is a vector?**
- **Is stress a scalar quantity?**
- **What is the difference between pressure and stress?**
- **What about pressure at a point inside a gas or a liquid will that be a scalar or a vector?**
  Pressure is a scalar as in steady state it acts equally in all direction, but necessarily has a definite direction, perpendicular to the surface

Pressure is the external force per unit area shown in the first picture.
Force F_1 per unit area of the face , on which it acts normally, is equal to the Pressure on that face.

Similarly, we can calculate the pressure on other two faces as F_2/ A and F_3 / A

Stress is the **internal restoring force** acting per unit area.
The squeezed ball, shown in picture 2, when compressed restores its shape, once the squeezing force is released.
The internal restoring force, acting per unit area, gives the stress.

Is Stress a vector quantity?

Stress is not a vector quantity since, unlike a force, the stress cannot be assigned a specific direction. Stress is neither a scalar nor a vector quantity. It is a tensor. You will learn more about tensor in your advanced science courses.

STRAIN

Measuring the effect of deforming force?

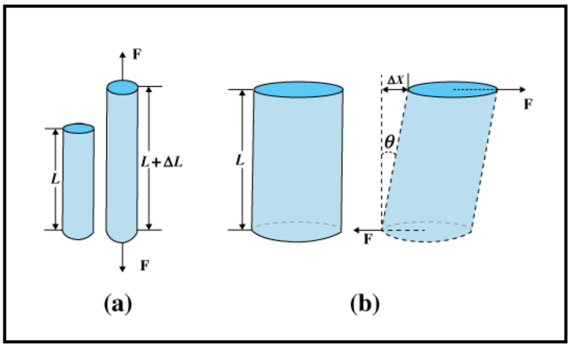
The application of deforming force may result in change in configuration of the body. The relative change, in size or shape of the body, is taken to measure its strain.

Strain is a dimensionless quantity.

The shape of the body and how the deforming force is applied to it, are important in determining strain. Wires, rods, beams have their length as much longer as compared to their other two dimensions (thickness, breadth). A flat sheet, or a three dimensional object, will be affected in a different way.

In order to make our study basic and simple, we will confine our study to solid objects with large length, for which we will describe longitudinal stress and longitudinal strain. We can also talk of volumetric stress and volumetric strain. In case the body is subjected to deforming forces that produce twist in the body, we need to talk of shear stress and strain.

There are **three ways** in which a solid may change its dimensions when an external force acts on it. These are shown in the figure below.



a) A cylinder is stretched by two equal forces applied normal to its cross-sectional area. The restoring force per unit area in this case is called tensile stress.

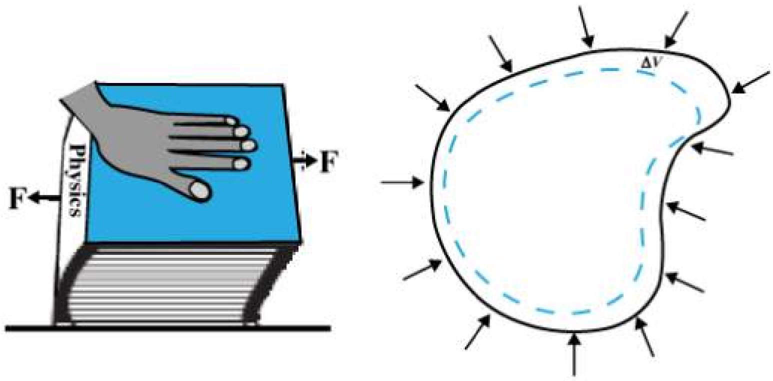
If the cylinder is compressed under the action of applied forces, the restoring force per unit area is known as compressive stress. Tensile, or compressive, stress can also be termed as longitudinal stress. In both the cases, there is a change in the length of the cylinder. The change in the length ΔL, to the original length L of the body (cylinder in this case), is known as longitudinal strain.

$$\text{Longitudinal strain} = \Delta L / L.$$

However, if two equal and opposite deforming forces are applied parallel to the cross-sectional area of the cylinder, (as shown in the fig.) there is relative displacement between the opposite faces of the cylinder. The restoring force per unit area, developed due to the applied tangential force is known as tangential or shearing stress.

b) As a result of the applied tangential force, there is a relative displacement Δx between opposite faces of the cylinder.

The strain so produced is known as shearing strain and it is defined as the ratio of relative displacement of the faces Δx to the length of the cylinder L. Shearing strain= Δ x/ L = tan θ , where θ is the angular displacement of the cylinder from the vertical (original position of the cylinder). Usually θ is very small and tan θ is nearly equal to angle θ, (if θ = 10°, for example, there is only 1% difference between θ and tan θ).



It can also be visualised, when a book is pressed with the hand and pushed horizontally, as shown in Fig.

Thus, shearing strain = tan θ ≈ θ

c) A solid sphere, placed in the fluid under high pressure is compressed uniformly on all sides.

The force applied by the fluid acts in perpendicular direction at each point of the surface and the body is said to be under hydraulic compression.

This leads to decrease in its volume without any change of its geometrical shape. The body develops internal restoring forces that are equal and opposite to the forces applied by the fluid (the body restores its original shape and size when taken out from the fluid).

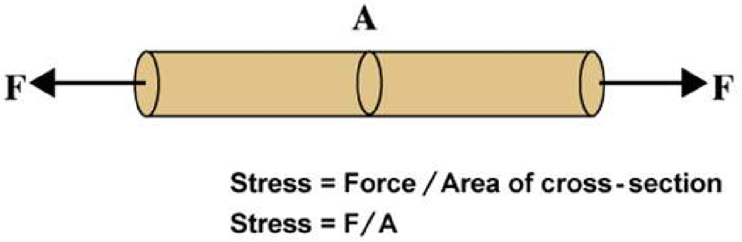
The internal restoring force per unit area, in this case, is known as hydraulic stress and its magnitude is equal to the hydraulic pressure (applied force per unit area). The strain, produced by a hydraulic pressure is called volume strain and is defined as the ratio of change in volume (ΔV) to the original volume (V).

Volume strain= Δ V/ V. Since the strain is a ratio of change in dimension to the original dimension, it has no units or dimensional formula.

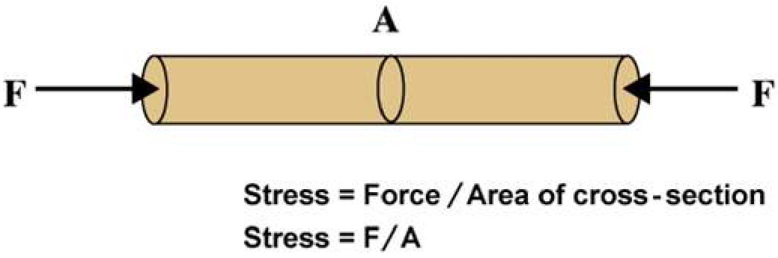
It is for this reason that scuba divers and deep sea divers wear special suits ,or else they would be crushed under huge hydrostatic deforming force.

7. TENSILE STRESS

Tensile means the material is under tension. A cylinder is stretched by two equal forces applied normal to its cross-sectional area. The restoring force per unit area in this case is called tensile stress.



If the cylinder is compressed under the action of applied forces, the restoring force per unit area is known as compressive stress.



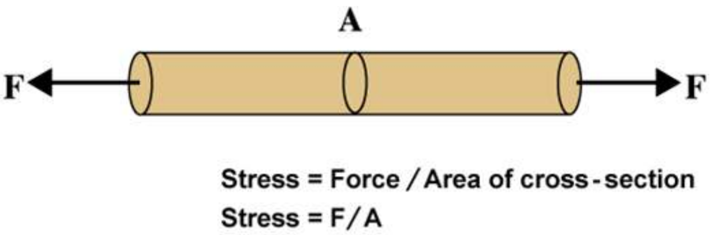
When an object is under tension (stretched state) it experiences an increase in length. For example: a rubber band being stretched.

The opposite of tension is compression, where an object is undergoing a decrease in length. For example: compressing a spring.

Longitudinal Stress and longitudinal strain

Tensile or compressive stress can also be termed as **longitudinal stress**, **because the force is applied along the length of the body.**

Longitudinal Stress = F / A
In case of a cylinder, the area of cross-section $A = \pi r^2$



Based on the manner in which external deforming forces are applied on the body, there exist different types of stress. We have talked about the longitudinal stress; the will be discussed later.

In both tensile and compressive stress, there is a change in the length of the cylinder. The change in the length ΔL, to the original length L of the body (cylinder in this case), is known as longitudinal strain.

It has no units as it is a ratio of two lengths.

Strain = $\frac{\Delta l}{L}$

Therefore, Longitudinal Stress = $\frac{F}{\pi r^2 L}$

Longitudinal Strain = $\frac{\Delta L}{L}$

8. RELATION BETWEEN LONGITUDINAL STRESS AND LONGITUDINAL STRAIN: Hooke's law

- **Robert Hooke is best known to physicists for his discovery of law of elasticity: "Ut tensio, sic vis" (This is a Latin expression and it means as the distortion, so the force). This law laid the basis for studies of stress and strain and for understanding the elastic properties of materials.**

For small deformations, (within a limit called elastic limit) the stress and strain are proportional to each other. This is known as Hooke's law.

Thus, stress is proportional to strain

stress α strain

stress = *k* × strain

where k is the proportionality constant; it is known as the **modulus of elasticity**.

Hooke's law is an empirical law and is found to be valid for most materials. However, there are some materials which do not exhibit this linear relationship.

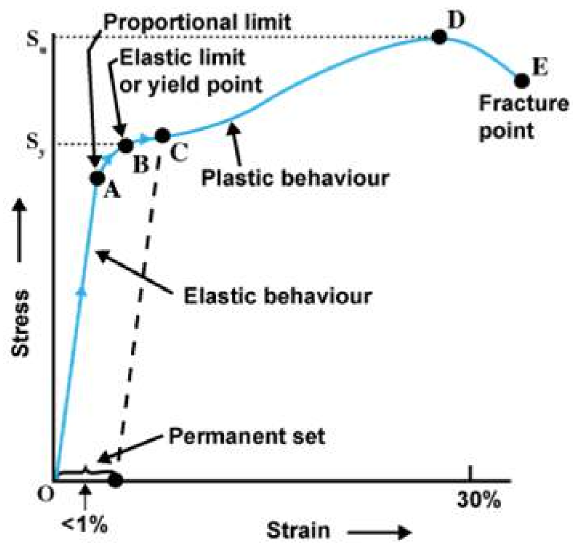
9. STRESS – STRAIN CURVE

The relation, between the stress and the strain for a given material, under tensile stress, can be found experimentally.

In a standard test of tensile properties, a test cylinder or a wire is stretched by applying a force.

The specimen (wire or cylindrical rod) is fixed at one end and tensile load is applied on the other end. The corresponding values, of load and extension in the rod, are noted down. The (relative) fractional change in length (the strain) is calculated.

The applied force is gradually increased in steps and the change in length is noted. A graph is plotted between the stress (which is equal in magnitude to the applied force per unit area) and the strain produced.



These curves help us to understand how a given material deforms with increasing loads.

(Analogous graphs for compression and shear stress may also be obtained.)

(i) **Linear Region:** From the above stress-strain curve for a metal wire, we can see that in the region between O to A, the curve is linear. In this region, Hooke's law is obeyed, i.e., stress α strain. Within this region, the body regains its original dimensions when the applied force is removed. In this region, the solid behaves as an elastic body.

(ii) **Elastic limit**: In the region from A to B, stress and strain are not proportional. However, when the load is removed in the region from A to B, the body still returns to its original dimension. The point B in the curve is known as yield point (also known as elastic limit) and the corresponding stress is known as yield strength (S_Y) of the material.

(iii) **Permanent Set**: If the load is increased further, the stress developed exceeds the yield strength and strain increases rapidly even for a small change in the stress. The portion of the curve between B and D shows this. When the load is removed, say at some point C between B and D, the body does not regain its original dimension. In this case, even when the stress is zero, the strain is not zero. The material is said to have a permanent set. The deformation is said to be plastic deformation.

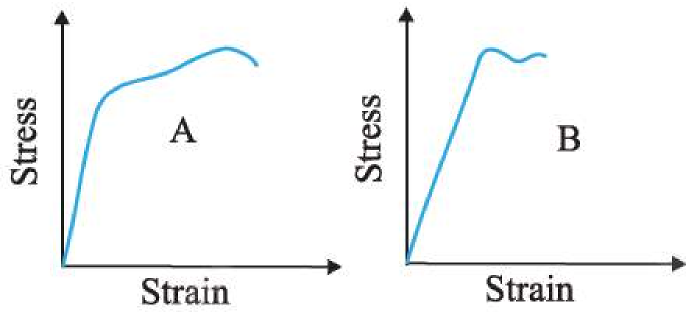
(iv) **Ultimate tensile strength:** The point D on the graph is the ultimate tensile strength (S_u) of the material. Beyond this point, additional strain is produced even by a reduced applied force and fracture occurs at point E.

(v) **Fracture or Breaking Point**: At this point E, the material fractures or breaks.

**If the ultimate strength and fracture points D and E are close to each other, the material is said to be brittle. If they are far apart, the material is said to be ductile.

EXAMPLE

Identify, from the graphs A and B shown below, the ductile or brittle nature of the material.



The graphs are drawn to the same scale.
(a) Which of the materials has a greater value for its Young's modulus?
(b) Which of the two is the stronger material?

SOLUTION

Material A is ductile while material B is likely to be brittle.

a) Y is indicated by the slope of stress strain graph within elastic limit. Hence material A has a greater value for its Young's modulus.
b) Material A as it can withstand a greater deformity.

EXAMPLE

Read the following two statements below carefully and state, with reasons, if it is true or false.
(a) The Young's modulus of rubber is greater than that of steel;
(b) Hooke's law is obeyed within elastic limit

SOLUTION

a) **False** we can easily break rubber bands with our hands by applying a small force but we cannot break a wire easily
b) **True**

As stated earlier, the stress-strain behaviour varies from material to material.

For example, rubber can be pulled to several times its original length and still returns to its original shape.

EXAMPLE

Four identical hollow cylindrical columns of mild steel support a big structure of mass 50,000 kg. The inner and outer radii of each column are 30 and 60 cm respectively. Assuming the load distribution to be uniform, calculate the compressional strain of each column.

SOLUTION:

Given: mass of structure (M) = 50,000 Kg

Inner radius of column (r) = 30cm = 0.3m

Outer radius of column (R) = 60cm = 0.6m

We have to find: compressional strain of each column $\left(\frac{\Delta L}{L}\right)$ =?

Young's modulus of rigidity (Y) = $\frac{MgL}{A\Delta L}$

And, for steel Y is = $2\times10^{11}Nm^{-2}$

$$\left(\frac{\Delta L}{L}\right) = \frac{Mg}{AY}$$

$$= \frac{Mg}{4\pi(R^2 - r^2)}$$

$$= \frac{50000\times9.8}{4\times3.14(0.6^2-0.3^2)}$$

$$\left(\frac{\Delta L}{L}\right) = 7.2\times10^{-7}$$

EXAMPLE

A piece of copper having a rectangular cross-section of 15.2 mm × 19.1 mm is pulled in tension with 44,500 N force, producing only elastic deformation. Calculate the resulting strain?

SOLUTION:

Given: area of cross-section of piece of paper (A) = 15.2mm×19.1mm

$= 15.2\times10^{-3}\times19.1\times10^{-3}m^2$

$= 2.9\times10^{-4}m^2$

Force of tension (F) = 44500N

To find: strain = $\frac{\Delta L}{L}$

Modulus of elasticity of copper (η) = 42×10^9 N/m^2

Modulus of elasticity (η) = $\frac{stress}{strain}$

$$= \frac{F/A}{\Delta L/L}$$

$$\frac{\Delta L}{L} = \frac{F}{A\eta}$$

$$= \frac{44500}{2.9\times10^{-4}\times42\times10^9}$$

$$\frac{\Delta L}{L} = 3.65\times10^{-3}$$

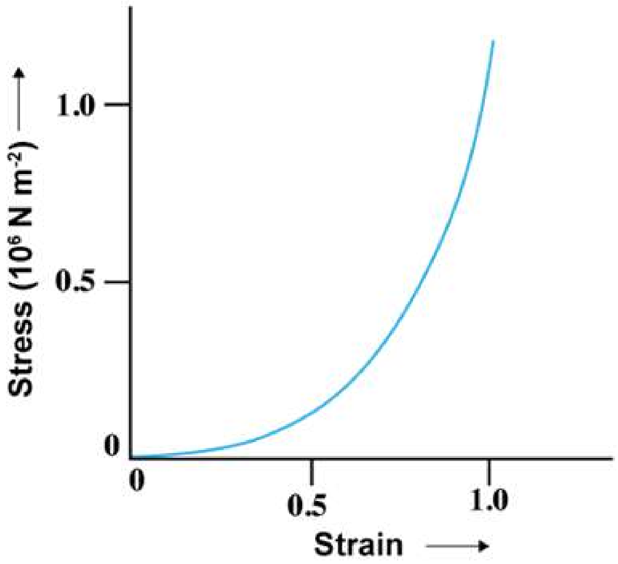
strain = 3.65×10^{-3}

TRY THESE

- **Why is the ratio of stress to the strain beyond the elastic limit not constant?**
- **Suppose we have a copper wire and we stretch it and there is an increase in its length , on removing the stretching force the wire does not regain its original length . Can we calculate Young's modulus in this situation?**
  - **If we have two wires of same length L and same cross-section area A, but to increase their length by 0.1cm, the force required is 450N and 550N for the first and second respectively. Then which one of the two would be more elastic?**

A class of solids, called elastomers, does not obey Hooke's law.

Stress-strain curve for the elastic tissue of aorta, present in the heart, has the slope shown below. As seen there is no linear region in this graph. Although elastic region is very large, the material does not obey Hooke's law over most of the region. Secondly, there is no well-defined plastic region. Substances, like tissue of aorta, rubber etc., which can be stretched to cause large strains, are called elastomers.



10. YOUNG'S MODULUS ('Y') OF ELASTICITY

The proportional region, within the elastic limit of the stress-strain curve (region OA in stress-strain curve of a metal/ ductile material), is of great importance for structural and manufacturing engineering designs.

The ratio of stress and strain, called modulus of elasticity, is found to be a characteristic of the material.

Experimental observation show that for a given material, the magnitude of the strain produced is same whether the stress is tensile or compressive. The ratio of tensile (or compressive) stress (σ) to the longitudinal strain (ε) is known as the Young's modulus of the material; it is denoted by the symbol Y.

We have: $Y = \frac{\frac{F}{A}}{\frac{\Delta L}{L}}$

$\therefore Y = (F/A)/(\Delta L/L) \quad = (F \times L)/(A \times \Delta L)$

Since strain is a dimensionless quantity, the unit of Young's modulus is the same as that of stress i.e., N m^{-2} or pascal (Pa).

This Table below gives the values of Young's moduli and yield strengths of some materials.

Table Young's moduli and yield strengths of some materials.

Substance	Density ρ (kg m^{-3})	Young's modulus Y (10^9 N m^{-2})	Ultimate strength, S_u (10^6 N m^{-2})	Yield strength S_y (10^6 N m^{-2})
Aluminium	2710	70	110	95
Copper	8890	110	400	200
Iron (wrought)	7800-7900	190	330	170
Steel	7860	200	400	250
Glass#	2190	65	50	—
Concrete	2320	30	40	—
Wood#	525	13	50	—
Bone#	1900	9	170	—
Polystyrene	1050	3	48	—

 # substance tested under compression

We can notice that **for metals Young's moduli are large**. Therefore, these materials require a large force to produce a small change in their length.

The value of Young's modulus is maximum for steel and thus steel is more elastic in comparison to other materials mentioned in the table. For the same change in length, steel requires a large amount of force.

To increase the length of a thin steel wire of 0.1 cm^2 cross-sectional area by 0.1%, a force of 2000 N is required.

The force required to produce the same strain in aluminium, brass and copper wires, having the same cross-sectional area, are 690 N, 900 N and 1100 N, respectively.

It means that steel is more elastic than copper, brass and aluminium. It is for this reason that steel is preferred in heavy-duty machines and in structural designs. Wood, bone, concrete and glass have rather small values for their Young's moduli.

The Young's modulus is relevant only for solids since only solids can have well defined lengths and shapes.

Example

A structural steel rod has a radius of 10 mm and a length of 1.0 m. A 100 kN force stretches it along its length.

Calculate

(a) Stress

(b) Elongation

(c) Strain on the rod.

Young's modulus, for structural steel, is 2.0 × 10^{11} N m^{-2}.

SOLUTION

We assume that the rod is held by a clamp at one end, and the force F is applied at the other end, parallel to the length of the rod. Then the stress on the rod is given by

Stress = F/ A

= F /πr^2

= 3.18 × 10^8 N m^{-2}

The elongation ΔL, (from Y= (F/A)/(ΔL/L)), is given by ΔL= $\frac{F \times L}{A \times Y}$

= 1.59 mm

The strain is given by: Strain = ΔL/L = (1.59 × 10^{-3} m)/(1m)

= 1.59 × 10^{-3} = 0.16 %

EXAMPLE

A copper wire of length 2.2 m and a steel wire of length 1.6 m, both of diameter 3.0 mm, are connected end to end. When stretched by a load, the net elongation is found to be 0.70 mm. Obtain the load applied.

SOLUTION

The copper and steel wires are under equal tensile stress because they have the same tension (equal to the load W) and the same area of cross-section A.

We have stress = strain × Young's modulus

Therefore W/A = Yc × (ΔLc/Lc) = Ys × (ΔLs/Ls), where the subscripts c and s refer to copper and stainless steel respectively.

Or, ΔLc/ΔLs = (Ys/Yc) × (Lc/Ls)

Given Lc = 2.2 m, Ls = 1.6 m, the value of Yc = 1.1 × 10^{11} N m^{-2}, and

Ys = 2.0 × 10^{11} N m^{-2}

ΔLc/ΔLs = (2.0 × 10^{11}/1.1 × 10^{11}) × (2.2/1.6) = 2.5

The total elongation is given to be ΔLc + ΔLs = 7.0 × 10^{-4} m

Solving the above equations, ΔLc = 5.0 × 10^{-4} m, and ΔLs = 2.0 × 10^{-4} m

Therefore W = (A × Yc × ΔLc)/Lc = π (1.5 × 10^{-3})^2 × [(5.0 × 10^{-4} × 1.1 × 10^{11})/2.2] = 1.8 × 10^2 N

EXAMPLE

In a human pyramid in a circus, the entire weight of the balanced group is supported by the legs of a performer who is lying on his back. The combined mass of all the persons performing the act, and the tables, plaques etc. involved is 280 kg.

The mass of the performer lying on his back at the bottom of the pyramid is 60 kg. Each thighbone (femur) of this performer has a length of 50 cm and an effective radius of 2.0 cm. Determine the amount by which each thighbone gets compressed under the extra load.

SOLUTION

Total mass of all the performers the tables and plaques = 280 kg

Mass of the performer = 60 kg

Mass supported by the legs of the performer at the bottom of the pyramid = (280-60) kg = 220 kg

Weight of this supported mass = 220 kg wt. = 220 × 9.8 N = 2156 N

Weight supported by each thighbone of the performer = ½ (2156) N = 1078 N

From Table 9.1 of NCERT the Young's modulus for bone is given by:

Y = 9.4 × 10^9 Nm^{-2}

Length of each thighbone L = 0.5 m

The radius of thighbone = 2 cm

Thus the cross sectional area of the thighbone

A = π× (2×10^{-2})^2 m^2 = 1.26 × 10^{-3} m^2)

Using equation

Δ L = [(F ×L)/ (Y×A)]

= [(1078×0.5)/(9.4 ×10^9×1.26×10^{-3})]

= 4.55 ×10^{-5} m or 4.55 ×10^{-3} cm

This is a very small change! The fractional decrease in the thighbone is Δ L/L = 0.000091

Or 0.0091%

EXAMPLE

A steel cable, with a radius of 1.5 cm, supports a chairlift at a ski area. If the maximum stress is not to exceed 10^8 N m^{-2}, what is the maximum load the cable can support?

SOLUTION

Maximum stress = $\frac{Maximum\ load}{area\ of\ cross-section}$

∴ Maximum load = 7.07 ×10^4 N

EXAMPLE

A rigid bar, of mass 15 kg, is supported symmetrically by three wires each 2.0 m long. Those at each end are of copper and the middle one is of iron. Determine the ratios of their diameters if each is to have the same tension.

SOLUTION:

As the bar is supported symmetrically by three wires, T and ΔL in each is same, given L same.

Therefore, Y α $\frac{1}{A}$

Area = $\frac{\pi D^2}{4}$, $\frac{D_{Cu}}{D_{Fe}} = \sqrt{\frac{Y_{Fe}}{Y_{Cu}}}$ = 1.3

NOW TRY THESE

- Why are springs made of steel and not of copper?
- Is rubber more elastic than steel?
- Two identical solid balls, one of rubber and the other of wet clay are dropped from the same height on a concrete floor which will rise to a greater height on striking the floor and why ?
- Identical springs of steel and copper are equally stretched, on which more work will have to be done

  Y of steel is greater than Y of copper, hence for same strain a larger force will have to be applied on the steel spring than that of copper steel, hence more work will have to be done on steel spring.
- If deforming force is applied to a body but its size do not change then what we will call this body-
  - a. Elastic
  - b. Plastic
  - c. Rigid
  - d. Such body is not possible
- Which body will be more elastic-
  - a. One which require larger force to increase the same length
  - b. One, in which increase in length will be greater for the same force applied
  - c. Both one and two
  - d. None on these

11. SUMMARY

In this unit you have learnt the following

- Solids tend to regain their original shape and size once external deforming force is removed
- The property of solids by virtue of which they regain their original shape and size even when the deforming forces are removed is called **elasticity**

- The property of solids by virtue of which they do not regain their original shape and size, even when the deforming forces are removed, is called **plasticity**.
- Beyond a limit, called **elastic limit,** the solid does not regain its original shape and size
- External deforming force may cause change in length, volume or shape
- A restoring force develops within the solid opposing the deforming force
- The restoring force, developed per unit area is called **stress**. We have longitudinal, or tensile stress, in case the length of the solid (like a wire or a rod), has a value much greater than its other two dimensions.
- Volumetric stress develops in solids if the deformation is caused due to hydrostatic forces
- Shear stress is associated with change in shape
- **Strain** is a measure of the relative deformity

  Longitudinal strain = change in length /original length = $\frac{\Delta l}{L}$
- Volumetric strain = change in volume /original volume
- Shear strain is measured in terms of angular or lateral deformation
- **Hooke's law** states that stress is proportional to strain within the elastic limit.
- Modulus of elasticity is the ratio of stress to strain
- **Young's' modulus** Y = longitudinal stress /longitudinal strain
- The stress strain graph shows the elastic and plastic behaviour of solids